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# Inclusive Decays of $B$ Mesons into $D_s$ and $D_s^*$ at $O(\alpha_s)$ Including $D_s^*$ Polarization Effects

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## Abstract

The dominant contribution to the inclusive decays of  $B$  mesons into the charm-strangeness mesons  $D_s$  and  $D_s^*$  is expected to be given by the partonic process  $b \rightarrow c + (D_s^-, D_s^{*-})$ . We determine the nonperturbative  $O(1/m_b^2)$  and the  $O(\alpha_s)$  radiative corrections to  $b \rightarrow c + (D_s^-, D_s^{*-})$  and thereby to the inclusive decays  $\bar{B} \rightarrow X_c + (D_s^-, D_s^{*-})$ . The new feature of our calculation is that we separately determine the nonperturbative and the  $O(\alpha_s)$  corrections to the longitudinal ( $L$ ) and transverse ( $T$ ) pieces of the spin 1  $D_s^{*-}$  meson. The longitudinal/transverse composition of the  $D_s^{*-}$  can be probed through its two principal decay modes  $D_s^{*-} \rightarrow D_s^- + \gamma$  and  $D_s^{*-} \rightarrow D_s^- + \pi^0$  for which we write down the angular decay distributions.

# 1 Introduction

In a recent paper Aleksan et al. have convincingly argued that the inclusive decay  $\bar{B} \rightarrow X_c + (D_s^-, D_s^{*-})$  is dominated by the partonic process  $b \rightarrow c + (D_s^-, D_s^{*-})$  [1]. The basic assumption is that factorization holds for the nonleptonic decay process  $\bar{B} \rightarrow X_c + (D_s^-, D_s^{*-})$ . One can then factorize the transition into a current-induced  $\bar{B} \rightarrow X_c$  transition and a current-induced vacuum one-meson transition. The leading order contribution to the  $\bar{B} \rightarrow X_c$  transition is given by the partonic  $b \rightarrow c$  transition. Corrections to the leading order result set in only at  $O(1/m_b^2)$ . They can be estimated using the methods of the operator product expansion in HQET.

Aleksan et al. also pointed out that it would be interesting to experimentally measure the longitudinal/transverse composition of the spin 1 meson  $D_s^{*-}$  in this inclusive decay which they computed at the Born term level. In an accompanying paper the same authors calculated the  $O(\alpha_s)$  corrections to the inclusive rates into the spin 0  $D_s^-$  and the spin 0  $D_s^{*-}$  [2] without,<sup>1</sup> however, separating the longitudinal ( $L$ ) and transverse ( $T$ ) contributions in the spin 1 case. It is the purpose of this paper to fill the gap left by [2] and to provide analytical formulae as well as the relevant numerical results for the  $O(\alpha_s)$   $L/T$  content of the  $D_s^{*-}$  in this reaction. We emphasize that the  $O(\alpha_s)$  corrections calculated here and in [8] are only partial. There are also non-factorizing  $O(\alpha_s)$  corrections as Beneke et al. [9] and Chay [8] have explicitly shown for the exclusive decays  $\bar{B} \rightarrow \pi\pi$  and  $\bar{B} \rightarrow D^{(*)}\pi^-$ , respectively. The non-factorizing  $O(\alpha_s)$  corrections are colour suppressed and are thus expected to be small as e. g. explicitly shown for  $\bar{B} \rightarrow D^{(*)}\pi^-$  in [8]. As concerns the nonperturbative effects we also write down the  $L/T$  composition of the nonperturbative  $O(1/m_b^2)$  contribution to the  $D_s^*$  rate as well as to the spin 0  $D_s$  rate using results of [10].

At the Born term level Aleksan et al. found  $\Gamma_L/\Gamma_T = 1.823$  (using their mass values  $m_b = 4.85$  GeV and  $m_c = 1.45$  GeV). It would be interesting to see how radiative corrections and the nonperturbative contributions affect this ratio. In the corresponding case  $t \rightarrow b + W^+$  (with  $m_b = 0$ ) we found earlier that the ratio  $\Gamma_L/\Gamma_T$  is shifted downward by the  $O(\alpha_s)$  radiative corrections by an amount of 3.5% [11].

## 2 Angular decay distributions

The longitudinal and transverse content of the diagonal density matrix of the  $D_s^{*-}$  (or its charge conjugate state  $D_s^{*+}$ ) can be determined by analysing the angular decay distribution of its subsequent decay into  $D_s^{*-} \rightarrow D_s^- + \gamma$  and  $D_s^{*-} \rightarrow D_s^- + \pi^0$ . The branching ratios into these two principal channels are given by  $(94.2 \pm 2.5)\%$  and

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<sup>1</sup>The  $O(\alpha_s)$  corrections to the spin 1 piece of the weak current keeping both quark masses finite had been calculated before in [3–5]. The  $O(\alpha_s)$  corrections to the spin 0 piece of the weak current can be deduced from the corresponding calculation for  $t \rightarrow b + H^+$  [6]. Latter result had also been used in a calculation of the  $O(\alpha_s)$  radiative corrections to  $b \rightarrow c + \tau^- + \bar{\nu}_\tau$  [7] where the spin 0 piece enters because the  $\tau$ -mass cannot be neglected in this process.

$(5.8 \pm 2.5)\%$  [12], respectively. In terms of the diagonal density matrix elements  $\rho_{mm}$  ( $m=0(L), \pm 1(T)$ ) of the  $D_s^{*-}$  the polar angle distribution is given by

$$W(\theta) \propto \sum_{m,m'} \rho_{mm} d_{mm'}^{(1)}(\theta) d_{mm'}^{(1)}(\theta) |h_{m'}|^2. \quad (1)$$

The  $h_m$  are the decay amplitudes of the decays  $D_s^{*-} \rightarrow D_s^- + \gamma$  ( $m = \pm 1$ ) and  $D_s^{*-} \rightarrow D_s^- + \pi^0$  ( $m = 0$ ) where the  $m$  are the magnetic quantum numbers of the  $D_s^{*-}$  in the decay frame. The  $d_{mm'}^{(1)}(\theta)$  are the usual Wigner  $d$ -function and  $\theta$  is the polar angle of the  $D_s^-$  in the  $D_s^{*-}$  rest frame (measured with regard to the original momentum direction of the  $D_s^{*-}$ ) as shown in Fig. 1. One thus obtains the polar angle decay distributions

$$\frac{d\Gamma_{\bar{B} \rightarrow X_c + D_s^{*-} (\rightarrow D_s^- + \gamma)}}{d\cos\theta} = \text{BR}(D_s^{*-} \rightarrow D_s^- + \gamma) \left( \frac{3}{8}(1 + \cos^2\theta)\Gamma_T + \frac{3}{4}\sin^2\theta\Gamma_L \right) \quad (2)$$

and

$$\frac{d\Gamma_{\bar{B} \rightarrow X_c + D_s^{*-} (\rightarrow D_s^- + \pi^0)}}{d\cos\theta} = \text{BR}(D_s^{*-} \rightarrow D_s^- + \pi^0) \left( \frac{3}{4}\sin^2\theta\Gamma_T + \frac{3}{2}\cos^2\theta\Gamma_L \right). \quad (3)$$

Considering the fact that the upcoming  $B$ -factories will be producing upward of 10K  $B\bar{B}$  pairs per day and that the inclusive branching ratio of the  $B$ 's into  $D_s^{*-}$ 's is expected to lie around  $O(5\%)$  it should not be too difficult to experimentally determine the angular coefficients of the two decay distributions and thereby the  $L/T$  content of the  $D_s^*$ .

### 3 Born term rates and $O(\alpha_s)$ radiative corrections

Let us begin by writing down the Born term level results for  $b \rightarrow (D_s^-, D_s^{*-}) + c$  (see Fig. 2a). We shall closely follow the notation of Aleksan et al. [1] throughout. For easy comparison with the numerical results of [1] we shall also adhere to their numerical parameter values. One has

$$\Gamma_S^{(0)}(b \rightarrow D_s^- + c) = \frac{G_F^2}{8\pi} |V_{bc} V_{cs}^*|^2 f_{D_s}^2 \frac{(m_b^2 - m_c^2)^2}{m_b^2} \left( 1 - \frac{m_{D_s}^2 (m_b^2 + m_c^2)}{(m_b^2 - m_c^2)^2} \right) p_{D_s} a_1^2, \quad (4)$$

$$\Gamma_{L+T}^{(0)}(b \rightarrow D_s^{*-} + c) = \frac{G_F^2}{8\pi} |V_{bc} V_{cs}^*|^2 f_{D_s^*}^2 \frac{(m_b^2 - m_c^2)^2}{m_b^2} \left( 1 + \frac{m_{D_s^*}^2 (m_b^2 + m_c^2 - 2m_{D_s^*}^2)}{(m_b^2 - m_c^2)^2} \right) p_{D_s^*} a_1^2, \quad (5)$$

$$\Gamma_L^{(0)}(b \rightarrow D_s^{*-} + c) = \frac{G_F^2}{4\pi} |V_{bc} V_{cs}^*|^2 f_{D_s^*}^2 \frac{(m_b^2 - m_c^2)^2}{m_b^2} \left( 1 - \frac{m_{D_s^*}^2 (m_b^2 + m_c^2)}{(m_b^2 - m_c^2)^2} \right) p_{D_s^*} a_1^2. \quad (6)$$

In Eqs. (4-6)  $f_{D_s}$  and  $f_{D_s^*}$  denote the pseudoscalar and vector meson coupling constants defined by  $\langle D_s^- | A^\mu | 0 \rangle = i f_{D_s} p_{D_s}^\mu$  and  $\langle D_s^{*-} | V^\mu | 0 \rangle = f_{D_s^*} m_{D_s^*} \epsilon^{*\mu}$ , respectively. The

Kobayashi-Maskawa matrix element is denoted by  $V_{q_1 q_2}$  and the  $p_{D_s}$  and  $p_{D_s^*}$  are the three-momenta of the  $D_s$  and  $D_s^*$  in the  $b$  rest system. The parameter  $a_1$  is related to the Wilson coefficients of the renormalized current-current interaction and is obtained from a combined fit of several decay modes ( $|a_1| = 1.00 \pm 0.06$ ) [1]. Note that the structural similarity of the rate formulae for the decay into  $D_s$  and the longitudinal  $D_s^*$  is an accident of the Born term calculation and does not persist e.g. at higher orders of  $\alpha_s$ .

Using  $f_{D_s} = 230$  MeV and  $f_{D_s^*} = 280$  MeV as in [1],  $\tau_B = 1.6$  ps,  $V_{bc} = 0.04$ ,  $V_{cs} = 0.974$  and the central value for  $a_1$  one arrives at

$$\text{BR}_{b \rightarrow D_s^- + c} \cong 3.2\% \quad \text{BR}_{b \rightarrow D_s^{*-} + c} \cong 6.8\%. \quad (7)$$

Summing up the  $D_s$  and  $D_s^*$  modes one arrives at a branching ratio of 10% which is consistent with the measured value  $BR(B \rightarrow D_s^\pm X) = (10.0 \pm 2.5)\%$  [12] if one assumes that the above two rates saturate the inclusive rate into  $D_s^\pm$ .

Next we turn to the  $O(\alpha_s)$  radiative corrections. As explained in [2], the radiative gluon corrections connect only to the  $b$  and  $c$  legs of the parton decay process  $b \rightarrow (D_s, D_s^*) + c$  because of the conservation of colour (see Fig. 2b, 2c and 2d). As remarked on earlier the radiative corrections for the spin 1 piece are then identical to the radiative corrections calculated in [3–5] or in [13] where the process  $t \rightarrow W^+ + b$  was considered keeping  $m_b \neq 0$ . In [13] we separately computed the radiative corrections to longitudinal ( $L$ ) and transverse ( $T$ )  $W^+$ 's in the decay process and thus these results can directly be transcribed to the present case.

The two  $L$  and  $T$  pieces can be projected from the hadron tensor by use of the projection operators<sup>2</sup> [13] (we use  $T = (L + T) - L$  since  $(L + T)$  is simple compared to either  $L$  or  $T$ )

$$\mathbb{P}_{L+T}^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right), \quad (8)$$

$$\mathbb{P}_L^{\mu\nu} = \frac{q^2}{m_b^2 |\vec{q}|^2} \left( p_b^\mu - \frac{p_b \cdot q}{q^2} q^\mu \right) \left( p_b^\nu - \frac{p_b \cdot q}{q^2} q^\nu \right). \quad (9)$$

The scalar spin 0 piece can be obtained with the projector

$$\mathbb{P}_S^{\mu\nu} = \left( \frac{q^\mu q^\nu}{q^2} \right). \quad (10)$$

The four-momentum of either the  $D_s$  or the  $D_s^*$  is denoted by  $q_\alpha$ . The magnitude of the three-momentum of  $q_\alpha$  is given by  $|\vec{q}| = \sqrt{q^2 - q_0^2}$ . We shall not dwell much on the details of our calculation in this short communication but refer to [13] for technical details. Let it be said that we use a gluon mass regulator to regularize the infrared singularities differing from Aleksan et al. [2] who use dimensional regularisation instead.

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<sup>2</sup>Here we use the notation “ $T$ ” (“transverse”) rather than the notation “ $U$ ” (“unpolarized transverse”) used in [13].

We shall present our  $O(\alpha_s)$  results in a form where the respective Born terms  $\Gamma_i^{(0)}$  are factored out from the  $O(\alpha_s)$  result. Including the Born term and the nonperturbative  $O(1/m_b^2)$  contributions to be discussed in Sec.4 we write with  $\hat{\Gamma}_S := \Gamma_S/\Gamma_S^{(0)}$ ,  $\hat{\Gamma}_{L+T,L} := \Gamma_{L+T,L}/\Gamma_{L+T}^{(0)}$  and  $\hat{\Gamma}_S^{(0)} = \hat{\Gamma}_{L+T}^{(0)} = 1$ ,  $\hat{\Gamma}_L^{(0)} := \Gamma_L^{(0)}/\Gamma_{L+T}^{(0)}$

$$\hat{\Gamma}_i = \hat{\Gamma}_i^{(0)} (1 + C_F \frac{\alpha_s}{\pi} \tilde{\Gamma}_i + K_b a_i + G_b b_i), \quad (11)$$

where  $i=S, L+T, L$ .  $K_b$  and  $G_b$  are the expectation values of the kinetic energy and the chromomagnetic interaction of the heavy quark in the  $B$  meson, respectively.

To begin we list the reduced  $O(\alpha_s)$  rates  $\hat{\Gamma}_i$ . For the reduced scalar spin 0 rate  $\tilde{\Gamma}_S$  we obtain

$$\begin{aligned} \tilde{\Gamma}_S = 2\Sigma + 1 - \frac{3}{4} \frac{\lambda^{1/2}}{x^2} \ln(w_1 w_\mu) + \lambda^{-1/2} B_S^{-1} \left\{ \frac{3}{2} \frac{(1-y^2)\lambda^{3/2}}{x^2} \ln(y) + y(1-y)^2 \times \right. \\ (1-x+y)(1+x+y) \ln(w_1 w_\mu) - \frac{1}{4} y^2 (1-x^2)(1+y^2) \mathcal{R}_{(-2,-1)} + \frac{1}{4} (1+y^2) \times \\ \left. (1-x^2+3y^2) \mathcal{R}_{(-1,-1)} - \frac{3}{4} (1+y^2) \mathcal{R}_{(0,-1)} - \frac{1}{2} (1+y^2) y^2 \mathcal{S}_{(0,0)} + \frac{1}{2} (1+y^2) \mathcal{S}_{(1,0)} \right\}, \end{aligned} \quad (12)$$

where we have defined a Born term-like scalar rate by

$$B_S = (1-y^2)^2 - x^2(1+y^2). \quad (13)$$

The other variables and functions appearing in Eq. (12) are explained at the end of this section. For the total reduced spin 1 rate we obtain

$$\begin{aligned} \tilde{\Gamma}_{L+T} = 2\Sigma - \frac{1}{4} \frac{\lambda^{1/2}}{x^2} \ln(w_1 w_\mu) + \lambda^{-1/2} B_{L+T}^{-1} \left\{ \frac{1}{2} \frac{(1-y^2)\lambda^{3/2}}{x^2} \ln(y) + \frac{1}{2} (\lambda + 6x^2 y) \times \right. \\ (1-x+y)(1+x+y) \ln(w_1 w_\mu) - \frac{1}{4} y^2 (1-x^2)(1+2x^2+y^2) \mathcal{R}_{(-2,-1)} + \frac{1}{4} ((1+x^2 - \\ 2x^4) + (4-3x^2)y^2 + 3y^4) \mathcal{R}_{(-1,-1)} - \frac{1}{4} (3-2x^2+3y^2) \mathcal{R}_{(0,-1)} - \frac{1}{2} (1+2x^2+y^2) \times \\ \left. y^2 \mathcal{S}_{(0,0)} + \frac{1}{2} (1+2x^2+y^2) \mathcal{S}_{(1,0)} \right\}, \end{aligned} \quad (14)$$

with

$$B_{L+T} = (1-y^2)^2 + x^2(1-2x^2+y^2). \quad (15)$$

Finally, the longitudinal piece of the reduced spin 1 rate is given by

$$\begin{aligned} \tilde{\Gamma}_L = 2\Sigma - \frac{1}{4} \frac{\lambda^{1/2}}{x^2} \ln(w_1 w_\mu) + \lambda^{-1/2} B_L^{-1} \left\{ \frac{1}{2} \frac{(1-y^2)\lambda^{3/2}}{x^2} \ln(y) + y(1-y)^2 \times \right. \\ \left. (1-x+y)(1+x+y) \ln(w_1 w_\mu) - \frac{1}{4} (1-x^2)^3 (1+y^2) y^2 \mathcal{R}_{(-2,1)} + \frac{1}{4} (1-x^2) \times \right. \end{aligned}$$

$$\begin{aligned}
& ((1-x^2)^2 + (6+x^2-3x^4)y^2 + (5+3x^2)y^4)\mathcal{R}_{(-1,1)} - \frac{1}{4}((5-2x^2-7x^4+4x^6) + \\
& (12-33x^2+x^4)y^2 + (7+x^2)y^4)\mathcal{R}_{(0,1)} + \frac{1}{4}((7-31x^2+4x^4) + (10+x^2)y^2 + \\
& 3y^4)\mathcal{R}_{(1,1)} - \frac{3}{4}(1+y^2)\mathcal{R}_{(2,1)} - \frac{1}{2}((1+10x^2-11x^4) + (1+x^2)^2y^2)y^2\mathcal{S}_{(0,2)} + \\
& \frac{1}{2}((1+10x^2-11x^4) + (3-4x^2+x^4)y^2 + 2(1+x^2)y^4)\mathcal{S}_{(1,2)} - \frac{1}{2}((2-6x^2) + \\
& (3+2x^2)y^2 + y^4)\mathcal{S}_{(2,2)} + 2(1+y^2)\mathcal{S}_{(3,2)} \Big\}, \tag{16}
\end{aligned}$$

where

$$B_L = B_S = (1-y^2)^2 - x^2(1+y^2). \tag{17}$$

The contribution denoted by  $\Sigma$  is the finite remainder of the Born term type one-loop contribution plus the soft gluon contribution. It is given by

$$\begin{aligned}
\Sigma = & \frac{1-x^2+y^2}{\lambda^{1/2}} \left\{ \text{Li}_2\left(1-\frac{w_1}{w_\mu}\right) - \text{Li}_2(1-w_1^2) - \text{Li}_2(1-w_1w_\mu) + \frac{1}{8} \ln\left(\frac{w_\mu}{w_1}\right) \ln\left(\frac{w_\mu}{w_1^3}\right) - \right. \\
& \frac{1}{4} \ln(w_1w_\mu) \left[ \ln\left(\frac{\lambda^{3/2}w_1^3(w_\mu-w_1)}{xy^2}\right) + 1 \right] - \ln w_1 \ln\left(\frac{1-w_1^2}{w_\mu-w_1}\right) \Big\} - \frac{1-y^2}{4x^2} \ln y + \\
& 1 - \frac{1}{2} \ln\left(\frac{\lambda^2}{x^2y^3}\right) + \frac{1}{4} \left[ \frac{\lambda^{1/2}}{2x^2} + \frac{x^2-2y}{\lambda^{1/2}} \right] \ln(w_1w_\mu) - \frac{1-y^2}{4\lambda^{1/2}} \ln\left(\frac{w_1^3}{w_\mu}\right), \tag{18}
\end{aligned}$$

where we use the abbreviations

$$w_1 := \frac{(1-x^2+y^2-\lambda^{1/2})x}{(1+x^2-y^2+\lambda^{1/2})y}, \quad w_\mu := \frac{(1-x^2+y^2-\lambda^{1/2})x}{(1+x^2-y^2-\lambda^{1/2})y}, \tag{19}$$

and  $x = m_{D_s^{(*)}}/m_b$  and  $y = m_c/m_b$ . The kinematical factor  $\lambda$  is defined by  $\lambda = 1+x^4+y^4-2x^2-2y^2-2x^2y^2$  such that  $p_{D_s^{(*)}} = \frac{1}{2}m_b\lambda^{1/2}$ .

The reduced  $O(\alpha_s)$  rates are given in terms of a set of tree graph phase space integrals  $\mathcal{R}_{(m,n)}$  and  $\mathcal{S}_{(m,n)}$  which are defined by

$$\mathcal{R}_{(m,n)} := \int_{y^2}^{(1-x)^2} \frac{z^m}{\lambda_z^{n/2}} dz, \quad \mathcal{S}_{(m,n)} := \int_{y^2}^{(1-x)^2} \frac{z^m}{\lambda_z^{n/2}} \ln\left(\frac{1-x^2+z+\lambda_z^{1/2}}{1-x^2+z-\lambda_z^{1/2}}\right) dz, \tag{20}$$

where  $\lambda_z = 1+x^4+z^2-2x^2-2z-2zx^2$ . Their solution can be obtained using techniques similar to the ones discussed in [14]. With the three abbreviations

$$\mathcal{N}_1 := \text{Li}_2(ux) - \text{Li}_2\left(\frac{x}{u}\right), \tag{21}$$

$$\mathcal{N}_2 := -\ln u \ln(1+x) + \ln\left(\frac{u-x}{(u-1)(1+x)}\right) \ln\left(\frac{u-x}{u(1-ux)}\right) + \tag{22}$$

$$\begin{aligned}
& - \operatorname{Li}_2\left(\frac{1}{u}\right) + \operatorname{Li}_2\left(\frac{u^2-1}{u(u-x)}\right) + \operatorname{Li}_2\left(\frac{1-ux}{u-x}\right), \\
\mathcal{N}_3 := & -\ln u \ln(1-x) - \ln\left(\frac{(u+1)(1-x)}{u-x}\right) \ln\left(\frac{u-x}{u(1-ux)}\right) + \\
& - \operatorname{Li}_2\left(-\frac{1}{u}\right) + \operatorname{Li}_2\left(\frac{(u^2-1)x}{u-x}\right) + \operatorname{Li}_2\left(-\frac{1-ux}{u-x}\right),
\end{aligned} \tag{23}$$

one has

$$\mathcal{S}_{(0,0)} = \lambda^{1/2} - 2x^2 \ln u - y^2 \ln\left(\frac{u-x}{u(1-ux)}\right), \tag{24}$$

$$\mathcal{S}_{(1,0)} = \frac{1}{4}(1+5x^2+y^2)\lambda^{1/2} - x^2(2+x^2)\ln u - \frac{1}{2}y^4 \ln\left(\frac{u-x}{u(1-ux)}\right), \tag{25}$$

$$\mathcal{S}_{(0,2)} = -\frac{1}{2x}(\mathcal{N}_2 - \mathcal{N}_3), \tag{26}$$

$$\mathcal{S}_{(1,2)} = -\frac{(1+x)^2}{2x}\mathcal{N}_2 + \frac{(1-x)^2}{2x}\mathcal{N}_3 + \mathcal{N}_1, \tag{27}$$

$$\begin{aligned}
\mathcal{S}_{(2,2)} = & -\frac{(1+x)^4}{2x}\mathcal{N}_2 + \frac{(1-x)^4}{2x}\mathcal{N}_3 + 2(1+x^2)\mathcal{N}_1 + \lambda^{1/2} - 2x^2 \ln u + \\
& - y^2 \ln\left(\frac{u-x}{u(1-ux)}\right),
\end{aligned} \tag{28}$$

$$\begin{aligned}
\mathcal{S}_{(3,2)} = & -\frac{(1+x)^6}{2x}\mathcal{N}_2 + \frac{(1-x)^6}{2x}\mathcal{N}_3 + (3+x^2)(1+3x^2)\mathcal{N}_1 + \frac{1}{4}(9+13x^2+y^2)\lambda^{1/2} + \\
& - (6+5x^2)x^2 \ln u + \frac{y^2}{2}(4(1+x^2)-y^2) \ln\left(\frac{u-x}{u(1-ux)}\right).
\end{aligned} \tag{29}$$

The non-logarithmic integrals are given by

$$\mathcal{R}_{(-2,-1)} = \frac{1}{y^2}\lambda^{1/2} + \ln u - \frac{1+x^2}{1-x^2} \ln\left(\frac{u-x}{1-ux}\right), \tag{30}$$

$$\mathcal{R}_{(-1,-1)} = -\lambda^{1/2} - (1+x^2)\ln u + (1-x^2) \ln\left(\frac{u-x}{1-ux}\right), \tag{31}$$

$$\mathcal{R}_{(0,-1)} = \frac{1}{2}(1+x^2-y^2)\lambda^{1/2} - 2x^2 \ln u, \tag{32}$$

$$\mathcal{R}_{(-2,1)} = \frac{1}{(1-x^2)^2} \left\{ \frac{\lambda^{1/2}}{y^2} + \frac{1+x^2}{1-x^2} \ln\left(\frac{u-x}{1-ux}\right) \right\}, \tag{33}$$

$$\mathcal{R}_{(-1,1)} = \frac{1}{1-x^2} \ln\left(\frac{u-x}{1-ux}\right), \quad \mathcal{R}_{(0,1)} = \ln u, \quad \mathcal{R}_{(1,1)} = -\lambda^{1/2} + (1+x^2)\ln u, \tag{34}$$

$$\mathcal{R}_{(-2,1)} = -\frac{1}{2}(3+3x^2+y^2)\lambda^{1/2} + (1+4x^2+x^4)\ln u, \tag{35}$$

where  $u := \frac{1+x^2-y^2+\sqrt{\lambda}}{2x}$ .

## 4 Nonperturbative contributions

When one uses the operator product expansion in HQET one can determine the nonperturbative corrections to the leading partonic  $b \rightarrow c$  rate. The nonperturbative corrections set in at  $O(1/m_b^2)$  and arise from the kinetic energy and the chromomagnetic interaction of the heavy quark in the heavy hadron. The strength of the kinetic and chromomagnetic interactions are parametrized by the expectation values of the relevant operators in the  $\bar{B}$  system and are denoted by  $K_b$  and  $G_b$ , respectively. The nonperturbative contributions to the spin 0 and spin 1 rates including the L / T separation have been calculated in [10] and can be taken from there. One has

$$S : a_S = -1, \quad (36)$$

$$b_S = (B_S \lambda)^{-1} \left[ -(1-y^2)^3(1-5y^2) + x^2(3-7y^2-11y^4+15y^6) - x^4(7+10y^2+15y^4) + 5x^6(1+y^2) \right], \quad (37)$$

$$L : a_L = -1 - \frac{16}{3}x^2 B_L^{-1}, \quad (38)$$

$$b_L = (3B_L \lambda)^{-1} \left[ -3(1-y^2)^3(1-5y^2) - x^2(7-27y^2+65y^4-45y^6) + x^4(27+34y^2-45y^4) - x^6(17-15y^2) \right], \quad (39)$$

$$T : a_T = -1 + \frac{16}{3}x^2 B_T^{-1}, \quad (40)$$

$$b_T = 2x^2(3B_T \lambda)^{-1} \left[ (1-y^2)(5-4y^2+15y^4) + x^2(9+10y^2+45y^4) - x^4(29+45y^2) + 15x^6 \right], \quad (41)$$

where

$$B_T = B_{L+T} - B_L = 2x^2(1-x^2+y^2). \quad (42)$$

For our numerical evaluations we use  $K_b = 0.013$  and  $G_b = -0.0065$  as in [10].

## 5 Numerical results

Using  $m_b = 4.85$  GeV,  $m_c = 1.45$  GeV,  $m_{D_s} = 1968.5$  MeV and  $m_{D_s^*} = 2112.4$  MeV and  $\alpha_s(m_b) = 0.2$  we obtain for  $b \rightarrow c$

$$\hat{\Gamma}_S = (1 - 0.09638 - 0.013 + 0.00467), \quad (43)$$

$$\hat{\Gamma}_L = 0.6459 (1 - 0.1103 - 0.03413 + 0.00966), \quad (44)$$

$$\hat{\Gamma}_T = 0.3541 (1 - 0.1079 + 0.02553 - 0.02769), \quad (45)$$

$$\hat{\Gamma}_{L+T} = (1 - 0.1095 - 0.00859 - 0.01803). \quad (46)$$



The radiative corrections reduce the rates by about 10%, where the reduction is rather uniform for the four different rates. The nonperturbative corrections range from 0.5% for the chromomagnetic correction to  $\hat{\Gamma}_S$  to a maximal 3.4% for the kinetic energy correction to  $\hat{\Gamma}_L$  with no uniform pattern in their contributions. At the Born term level the transverse/longitudinal composition is given by  $\hat{\Gamma}_T/\hat{\Gamma}_L = 0.55$ . This ratio is shifted upward by the insignificant amount of 0.3% through the radiative corrections. Adding all corrections one finds a 3% reduction in the ratio.

For the  $b \rightarrow u$  transitions with  $m_u = 0$ , i.e.  $y = 0$  we have

$$\hat{\Gamma}_S = (1 - 0.1694 - 0.013 + 0.00751), \quad (47)$$

$$\hat{\Gamma}_L = 0.7250 (1 - 0.1777 - 0.02923 + 0.01414), \quad (48)$$

$$\hat{\Gamma}_T = 0.2750 (1 - 0.1150 + 0.02978 - 0.02348), \quad (49)$$

$$\hat{\Gamma}_{L+T} = (1 - 0.1605 + 0.00055 - 0.00934). \quad (50)$$

In the  $b \rightarrow u$  case the dominance of the longitudinal rate is more pronounced. At the Born term level one finds  $\Gamma_T/\Gamma_L = 2x^2 = 0.38$ . The radiative corrections are no longer as uniform as in the  $b \rightarrow c$  case. Whereas the radiative corrections to  $\hat{\Gamma}_S$ ,  $\hat{\Gamma}_L$  and  $\hat{\Gamma}_{L+T}$  amount to 16%–17%, the radiative correction to the transverse rate  $\hat{\Gamma}_T$  is only 11.5%. Thus the ratio  $\Gamma_T/\Gamma_L$  is shifted upward by 7.6% by the radiative corrections. Adding up all corrections one finds a 10.4% upward shift for this ratio. Let us mention that our  $O(\alpha_s)$  results on  $\Gamma_{L+T}$  and  $\Gamma_S$  numerically agree with the results of [2] for both the  $b \rightarrow c$  and  $b \rightarrow u$  transitions.

As emphasized in the introduction the conclusions drawn in this paper on the radiative corrections are tentative in as much as there are also nonfactorizing  $O(\alpha_s)$  contributions which have not been included in our analysis. Although the nonfactorizing  $O(\alpha_s)$  contributions are colour suppressed and thus expected to be small it would nevertheless be worthwhile to try and estimate the nonfactorizing  $O(\alpha_s)$  contributions along the lines of [8] and [9].

The last point we want to discuss are the inclusive decays  $\bar{B} \rightarrow X_C + (\pi^-, \rho^-)$  which can also be induced by the diagrams Fig. 2 when the  $c \rightarrow s$  transition in the upper leg is replaced by a  $u \rightarrow d$  transition. Using  $f_{\pi^-} = 132$  MeV,  $f_{\rho^-} = 216$  MeV and  $V_{ud} = 0.975$  one finds the Born term branching fractions  $BR_{b \rightarrow \pi^- + c} \cong 1.6\%$  and  $BR_{b \rightarrow \rho^- + c} \cong 4.6\%$ . In the latter case the rate is dominated by the longitudinal contribution since  $q^2 = m_\rho^2$  is not far from  $q^2 = 0$  where the rate would be entirely longitudinal. In fact one finds  $\Gamma_T/\Gamma_L = 0.067$ . It is important to note that the diagrams Fig. 2 are not the only mechanisms that contribute to the inclusive decays  $\bar{B} \rightarrow X_C + (\pi^-, \rho^-)$ . Additional  $\pi^-$  and  $\rho^-$  mesons can also be produced by fragmentation of the  $c$ -quark at the lower leg.<sup>3</sup> As concerns the  $\rho^-$  mesons resulting from the fragmentation process they would not

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<sup>3</sup>As concerns the inclusive decays  $\bar{B} \rightarrow X_C + (D_s^-, D_s^{*-})$  the possibility of producing extra  $D_s^-$  and  $D_s^{*-}$  mesons through fragmentation of the  $c$ -quark is ruled out for kinematic reasons.

be polarized along their direction of flight. This lack of polarization as compared to the strong polarization of the  $\rho$  mesons from the weak vertex could possibly be used to separate  $\rho^-$  mesons coming from the two respective sources.

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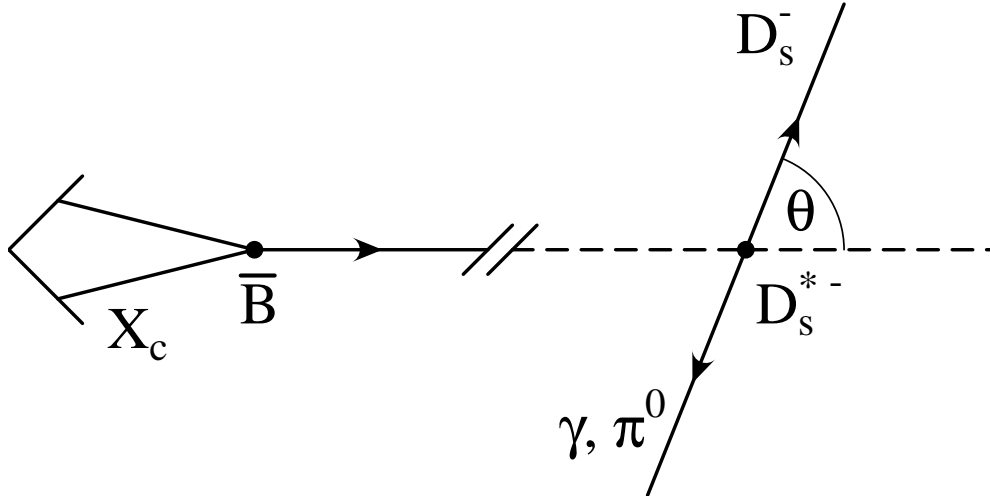


Figure 1: Definition of polar angle  $\theta$  in the inclusive decay  $\bar{B} \rightarrow X_c + D_s^* (\rightarrow D_s^- + \gamma \text{ or } \pi^0)$ . The polar angle  $\theta$  is defined in the  $D_s^{*-}$  rest frame relative to the direction of the  $D_s^{*-}$  in the  $\bar{B}$  rest frame.

**Figure 1**

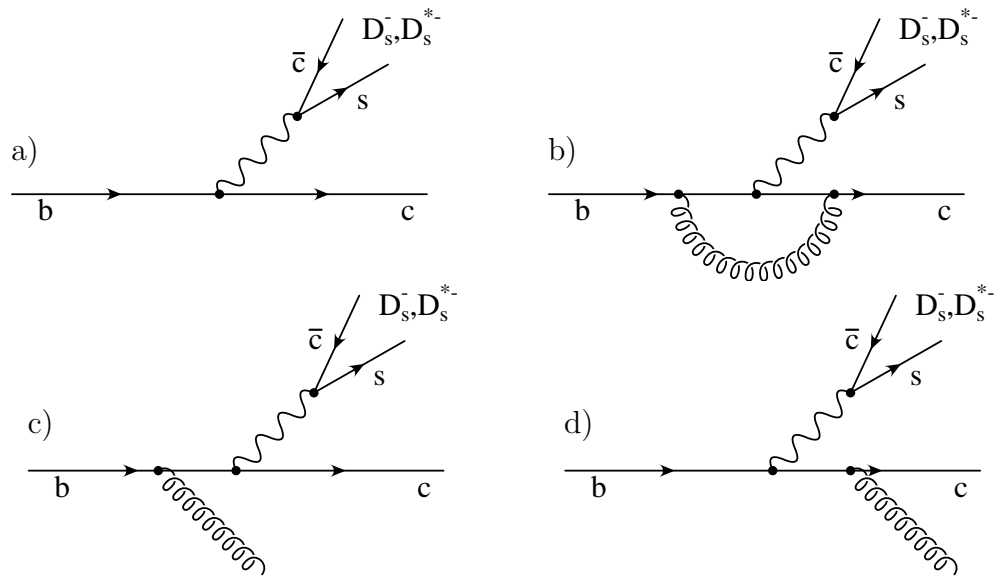


Figure 2: Leading order Born term contribution (a) and  $O(\alpha_s)$  contributions (b,c,d) to  $b \rightarrow c + (D_s^-, D_s^{*-})$ .

**Figure 2**

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